

MCEN90031

Applied High Performance Computing

Assignment 1

**The Shallow Water Equations**

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Master of Information Technology

26/09/2014

The Shallow Water equations describe the motion of a liquid when its depth is small compared to the wavelength of the surface waves. These equations derive from the principles of conservation of mass and conservation of momentum and give rise to a coupled system of first order nonlinear PDEs of the form:

(2)

(1)

Where:

**h** is the depth of the water [m],

**v** is the velocity of the fluid with components *v*x and *v*y [ms−1]

**g** is the gravitational constant 9.81N kg−1.

Computational domain (a square region of space) will be x ∈ [0, 100], y ∈ [0, 100], t ∈ [0, 100].

With periodic boundary conditions for all three fields and initial conditions:

vx(x, y, 0) = 0, vy (x, y, 0) = 0, h(x, y, 0) =

***Solve this system of equations using:***

* The Finite Difference method for the spatial discretization.
* Runge-Kutta method for the temporal discretization.

1. Perform a stability analysis on the fourth order Runge-Kutta method (applied to the model ODE problem), plotting the stability region, then perform an error analysis plotting the amplitude and phase errors as a function of λ∆t.

The first step is to translate the vector notation of the PDE’s.

We get the resulting matrix of the operation from equation (1):

Then the result is multiplied by the vector :

And following the equation (1) by using all the terms we get this:

Hence the equivalent PDE’s in 2D are:

(5)

(4)

(3)

The Second order central difference for the first derivative is:

The Fourth order central difference for the first derivative is:

Applying the second order central difference in the equations (3), (4) and (5) now we have:

Applying the fourth order central difference in the equations (3), (4) and (5) now we have: